### Results and Discussion

The static gas density measurements based on the intensity of the vibrational Q-branch of nitrogen are compared in Fig. 2 with the density values calculated from measured pressures and temperatures. The static density ($\rho$) is shown as a function of stagnation pressure ($P_o$). The Raman density values ($\rho_R$) are designated by symbols and the calculated density values ($\rho_c$) by line segments. The standard deviations of the Raman measurements lie within the symbols for this figure and are based on 10 sec of data. In general the agreement between the measured and calculated static density is good throughout the pressure range. The discrepancy at a 15° angle of attack is unexplained but may be associated with errors in calculated local density due to static pressure errors caused by outflow over the plate at this higher angle of attack.

The Raman measurements of static density and temperature obtained by using the pure rotational transitions are shown in Fig. 3. The Raman values of static density obtained from the intensity of the (6, 4) transition are plotted vs the calculated density values (Fig. 3a) and the Raman values of static temperature obtained from the intensities of the (6, 4) and (1, 3) transitions are shown vs the calculated temperatures values (Fig. 3b). The error bars designate one standard deviation based on 10-sec data points and these data were taken at the stagnation conditions $P_o = 3.45 \times 10^5$ N/m$^2$ and $T_o = 342$°K. The measured density obtained from the rotational transition (Fig. 3a) agrees as well with the calculated values as when the density was determined from the vibrational Q-branch approach. The measured static temperatures (Fig. 3b) also agree well with the calculated values.

### Conclusion

The present investigation has demonstrated the capability of the Raman scattering technique to provide local, nonintrusive measurements of static temperature and density in a hypersonic air facility. Static temperatures from 60° to 100°K and static densities from 0.03 to 0.8 Kg/m$^3$ were measured above a flat-plate model at Mach number 5 over a range of angles of attack (−5° to 15°) and stagnation conditions ($P_o = 1.7 \times 10^5$ to $2.8 \times 10^6$ N/m$^2$ and $T_o = 317°$ to 442°K). The measurement accuracy for static density and temperature of the present investigation shows that Raman scattering offers a viable approach to the measurement problems which exist for high-speed three-dimensional flows.

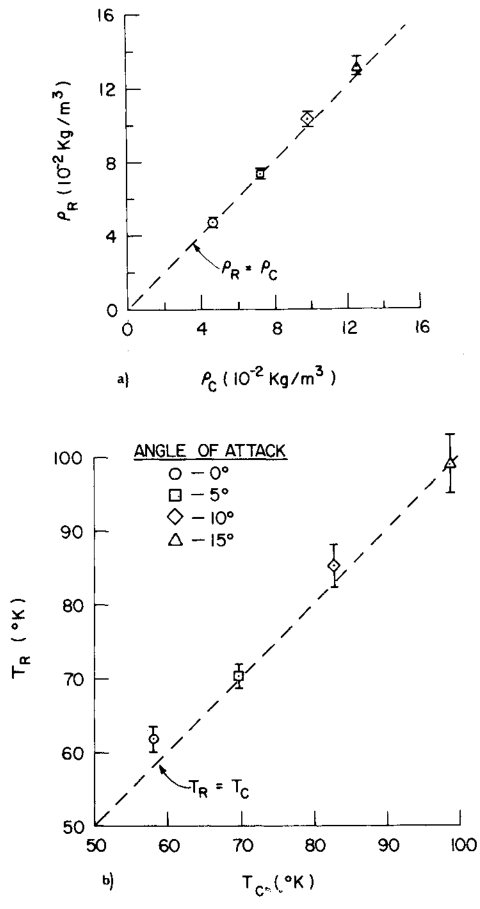


**Fig. 3 Static density (a) and temperature (b) comparison between measured rotational Raman values and calculated values.**

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# Optimal Scan of the Sky from a Rocket

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IN the context of making astronomical measurements by a rocket-launched instrument, McNutt has described a passive system which uses a resonant damper on a spinning rocket[1] to obtain an expanding scan of the sky. Though the resulting scan avoids the unwanted coverage of the ground obtained with simple roll and precession by Friedman and associates,[2] McNutt's scan provides a nonuniform scan rate as well as a nonuniform coverage of the sky. Furthermore, the orientation and pitch of his scan as well as the earlier ones depend upon the specific partition of the angular momentum between the two components. Inasmuch as the transverse component depends upon the time history of the asymmetry of the thrust from the engine as well as that of the aerodynamic restraint, this component is unknowable prior to the flight except in a statistical sense.

With the availability of a computer in an active control system,[3] it is possible in principle to generate any scan including one in which the center of the scan is the zenith, the scan rate is constant, the zenith distance of the axis of the instrument increases linearly with the azimuth distance of the axis of the instrument, and the scan axis is a fixed transverse axis of the instrument. These constraints, by assuring the maximum efficiency in the use of the instrument, a constant information rate from the instrument, a uniform coverage of the sky, and the freedom to use a noncircular field stop in the instrument, respectively, define the optimal scan. The purpose of this Note is to investigate the consequences of these constraints.

Let the instrument assembly be axisymmetric with subscripts $\lambda$ and $\tau$ connoting the axis of symmetry and the scan axis,

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respectively, and let $\omega$ be the angular velocity of the assembly. Then the components of $\omega$, $\omega_{\lambda,\tau}$, are the spin and cross spin, respectively. Nidey and Seames have shown[4] that the component of the external moment parallel to the cross spin of a rigid axisymmetric body modifies the cross spin directly independently of the spin, that the component normal to the cross spin as well as the axis of symmetry produces rotation of the cross spin and that the rate of rotation, $\Omega_\lambda$, and the latter component, $M_v$, are related by the following expression:

$$M_v = I_\tau \omega_\tau \Omega_\lambda - I_\lambda \omega_\lambda \omega_\tau \tag{1}$$

where $I_{\lambda,\tau}$ are the moments of inertia of the assembly. Since the scan axis is to be fixed in the assembly, $\Omega_\lambda = \omega_\lambda$ and

$$M_v = (I_\tau - I_\lambda)\omega_\tau \omega_\lambda \tag{2}$$

Furthermore, since $\omega_\tau$ is to be a constant

$$M_\tau = 0 \tag{3}$$

Note that to minimize $M_v$ the assembly should be spherically symmetrical. In this special case, $M_v$ as well as $M_\tau$ is zero and only a longitudinal control moment is required to produce an optimal scan.

Let $dR_\lambda \equiv \omega_\lambda dt$ and $dR_\tau \equiv \omega_\tau dt$ where $dt$ is the differential in time. Then $R_\lambda = \int \omega_\lambda dt$ and $R_\tau = \omega_\tau(t_f - t_i)$ where $t_{i,f}$ are the times the scan is initiated and finished, respectively. Since $N_v \equiv \int M_v dt$

$$N_v = (I_\tau - I_\lambda)\omega_\tau R_\lambda \tag{4}$$

and

$$t_f - t_i = R_\tau/\omega_\tau \tag{5}$$

where $t_f - t_i$ is the time required to execute the optimal scan and $N_v$ is the corresponding moment of the transverse impulse.

We can evaluate $R_{\lambda,\tau}$ by the use of the equations for a clelie[5] on a sphere of radius $R$. With changes in notation and corrections of several typographical errors, Loria's equations for the differential arc of the clelie, the geodesic curvature of the clelie and the angle between the arc of the clelie and the corresponding meridian, respectively, are as follows:

$$ds = Rd\theta(1+S^2)^{1/2} \tag{6}$$

$$\sigma = (C/R)(2+S^2)/(1+S^2)^{3/2} \tag{7}$$

and

$$\alpha = \arccos[-1/(1+S^2)^{1/2}] \tag{8}$$

where $S \equiv 2\pi \sin\theta/\Delta\theta$, $C \equiv 2\pi\cos\theta/\Delta\theta$, $\theta$ is the zenith distance of the clelie, and $\Delta\theta$ is the separation of the clelie, i.e., the increment in the zenith distance corresponding to an increase in the azimuth distance of $2\pi$. Since $dR_\tau = ds/R$, and $dR_\lambda = \sigma ds$

$$dR_\tau = d\theta(1+S^2)^{1/2} \tag{9}$$

$$dR_\lambda = d\theta(C)(2+S^2)/(1+S^2) \tag{10}$$

and

$$\omega_\lambda = \omega_\tau(C)(2+S^2)/(1+S^2)^{3/2} \tag{11}$$

Since $C = dS/d\theta$, we can integrate Eq. (10) directly giving $R_\lambda = [\arctan S + S]$. Thus we have

$$N_v = (I_\tau - I_\lambda)\omega_\tau[\arctan(2\pi\sin\theta/\Delta\theta) + 2\pi\sin\theta/\Delta\theta]_{\theta_i}^{\theta_f} \tag{12}$$

where $\theta_{i,f}$ are the initial and final zenith distances, respectively, and where this definite integral must be evaluated independently in each quadrant inasmuch as $N_v$, but not $R_\lambda$, is an unsigned scalar.

Though we cannot integrate the elliptic integral implicit in Eq. (9), we can simplify Eqs. (9) and (11) by making the following approximations:

$$dR_\tau \simeq d\theta(\sin\theta)(2\pi/\Delta\theta) \tag{13}$$

and

$$\omega_\lambda \simeq \omega_\tau/\tan\theta \tag{14}$$

respectively, with errors less than $\frac{1}{2}\%$ if $\Delta\theta < \pi/29$ and $\pi/18 < \theta < 17\pi/18$. Thus $R_\tau \simeq (2\pi/\Delta\theta)(-\cos\theta_f + \cos\theta_i)$

$$t_f - t_i \simeq (2\pi/\Delta\theta)|\cos\theta_i - \cos\theta_f|/\omega_\tau \tag{15}$$

and

$$M_v \simeq (I_\tau - I_\lambda)\omega_\tau^2/\tan\theta \tag{16}$$

to the same accuracy. Note that the exact expression for the latter is

$$M_v = (I_\tau - I_\lambda)\omega_\tau^2 C(2+S^2)/(1+S^2)^{3/2} \tag{17}$$

$$\leqq 4\pi(I_\tau - I_\lambda)\omega_\tau^2/\Delta\theta \tag{18}$$

since $C(2+S^2)/(1+S^2)^{3/2} \leqq 4\pi/\Delta\theta$; thus it is possible to scan to or from $\theta = 0$. Solving Eq. (15) for $\theta$

$$\theta \simeq \arccos[\cos\theta_i - \omega_\tau(t-t_i)/(2\pi/\Delta\theta)] \tag{19}$$

where the absolute value signs can be dropped if $\theta_f > \theta_i$. Hence

$$M_v \simeq (I_\tau - I_\lambda)\omega_\tau^2/\tan\{\arccos[\cos\theta_i - \omega_\tau(t-t_i)/(2\pi/\Delta\theta)]\} \tag{20}$$

By Eq. (8) we note that as $\theta \to 0$, $\alpha \to \pi$, and, if $\Delta\theta < \pi/29$ and $\pi/18 < \theta < 17\pi/18$

$$\alpha \simeq \arccos[-\csc\theta/(2\pi/\Delta\theta)] \tag{21}$$

with an error less than $\pi/6456$.

To put the preceding equations in perspective, let us adopt the parameters of the HI-HI-STAR instrument assembly which was to have been flown Jan. 26, 1974 from the White Sands Missile Range to an apogee of 275 km providing 393 sec above 100 km. These parameters are[6] $I_{\lambda,\tau} = 11$ and 374 kg-m$^2$, respectively, $M_v = 18.7$ n-m, and N < 3735 n-m-sec where the latter is the total moment of impulse available after the initial orientation of the assembly has been completed. Let $\omega_\tau$ by $\pi/15$ sec$^{-1}$ (12° sec$^{-1}$) and $\Delta\theta$ be $\pi/60$ (3°). Then by Eq. (17) the minimum zenith distance for which the transverse jets can produce an optimal scan is $0.225\pi$ (40.4°) and by Eq. (15) the zenith distance will have increased to $0.476\pi$ (85.7°) by the time the instrument has returned to 100 km from the same altitude. Exclusive of that required for roll control, by Eq. (12) the moment of impulse required to generate the scan would be 3183 n-m-sec. These numbers are quite reasonable. A different set of parameters and values may well be more appropriate for a specific scientific mission.

To execute an optimal scan, the control system would be used to establish the initial conditions $\theta_i$, $\phi_i$, $\alpha_i$, and $\omega_\tau$ whereupon $\theta$ would be controlled according to the law $\theta = (\Delta\theta/2\pi)\,\phi_t$ and $\alpha$, according to the law given in Eq. (8) where $\phi_{i,t}$ are the azimuth distances of the axis of the instrument at $t = t_i$ and $t$, respectively. Though these control laws require a computer, a programer will suffice if the lesser accuracy of an open loop scan is acceptable. In this event, after establishing the initial conditions, $M_v$ would be programed according to Eq. (20) while keeping the scan axis aligned with the cross spin. The moment of the impulse required for the closed loop scan will of course be greater than that for open loop, whereas the latter should be very close to that predicted by Eq. (12).

In any event, it is clear that an optimal scan of the sky is not impractical.

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